Navy Personnel Research and Development Center
San Diego, California 92152-7250 TN-96-25 March 1996

Nuclear Officer Retention:
An Economic Model

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
March 1996

3. REPORT TYPE AND DATE COVERED
Final--January 1995-January 1996

4. TITLE AND SUBTITLE
Nuclear Officer Retention: An Economic Model

5. FUNDING NUMBERS
Program Element: 0603707N
Work Unit: 0603707N.01770.MP111

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Navy Personnel Research and Development Center
53335 Ryne Road
San Diego, CA 92152-7250

8. PERFORMING ORGANIZATION REPORT NUMBER
NPRDC-TN-96-25

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
Functional Area: Workforce Management
Product Line: Force Management
Effort: Modelling Systems to Forecast the Effects of Compensation on Officer and Enlisted Retention Behavior

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE
A

13. ABSTRACT *(Maximum 200 words)*

From FY91 through FY94, the average retention rate at the end of the minimum service requirement (MSR) for nuclear-trained officers dropped to 60 percent from 70 percent, the average MSR retention rate from FY87 through FY90. To combat low retention in this community, the Nuclear Officer Incentive Pay (NOIP) program was designed, but its legislative authorization expires in FY96.

This report documents the results of an investigation of historical nuclear officer retention behavior at the end of their minimum service requirement. It specifies a 3-choice model and quantifies the impact of the NOIP retention bonus program on MSR retention.

Separate models of retention at MSR were estimated for the submarine and surface nuclear officer communities. For both communities, the retention elasticities with respect to the NOIP retention bonus program were small, but significant indicating the "pay does matter."

These models can be used to assess the retention and cost impacts of alternative NOIP retention strategies.

14. SUBJECT TERMS
Minimum service requirement (MSR), nuclear officer incentive pay (NOIP), continuation pay (COPAY), annual incentive bonus (AIB), elasticity, retention

15. NUMBER OF PAGES
26

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UNLIMITED

Foreword

This report was prepared as part of the New Modelling Systems to Forecast the Effects of Compensation on Officer and Enlisted Retention Behavior project (Program Element 0603707N), under the sponsorship of the Chief of Naval Personnel (PERS-2). The objective of this project is to quantify the effect of the Nuclear Officer Incentive Pay program on nuclear officer retention at the conclusion of their minimum service requirement. The work described here was performed in FY95 and FY96.

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Summary

Background

In today's Navy, nuclear-powered ships represent nearly 50 percent of the major combatants. Nuclear-trained officers, on the other hand, constitute only 16 percent of the Navy's unrestricted line officers. In recent years, the community has been plagued by accession shortfalls, high training attrition, and low retention. For example, from FY91 through FY94, the average percentage rate of officers staying in the Navy at the end of their minimum service requirement dropped to 60 percent from 70 percent, the average percentage rate from FY87 through FY90.

The Nuclear Officer Incentive Pay (NOIP) program was designed to combat low retention in this community, but its legislative authorization expires in FY96. To justify new authority for NOIP, current estimates of its effects on retention were needed.

Objective

The objective of this project was to quantify the effect of the NOIP program on nuclear officer retention at the end of their minimum service requirement (MSR).

Approach

The approach included: (1) developing a model of retention behavior at MSR, and (2) estimating and validating the model.

Results

Separate models of retention at MSR were estimated for the submarine and surface nuclear officer communities. For both communities, the retention elasticities with respect to the Continuation Pay (COPAY) and the Annual Incentive Bonus (AIB) were small. COPAY and AIB are the NOIP program's retention incentives. For submarine officers, the retention rate elasticity with respect to COPAY was .11. That is, for a 10 percent increase (decrease) in COPAY, a 1.1 percent increase (decrease) in the MSR retention rate for submarine officers was estimated. For surface warfare officers, the retention rate elasticity with respect to COPAY was .39. The retention rate elasticity with respect to AIB for submarine officers was .11; for surface warfare officers, it was .48.

Conclusions

Most retention-compensation research finds that "pay does matter." While the retention bonus elasticities found in these models are relatively small, their statistical significance also points to the fact that pay matters. These models can be used to assess the retention and cost impacts of alternative NOIP retention strategies.

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Introduction

In today's Navy, nuclear-powered ships represent nearly 50 percent of the major combatants. Nuclear-trained officers, on the other hand, constitute only 16 percent of the Navy's unrestricted line officers. Demanding scholastic requirements and attractive civilian opportunities have always made recruiting of qualified nuclear officer candidates difficult. From FY81 to FY94, the nuclear submarine officer community made its accession goal in only 3 of the 14 years. Moreover, from FY91 through FY94, the average percentage rate of officers staying in the Navy at the end of their minimum service requirement dropped to 60 percent from 70 percent, the average percentage rate from FY87 through FY90.

The Nuclear Officer Incentive Pay (NOIP) program was initiated in June 1969 to combat the chronic shortage of nuclear-trained submarine officers. Over the years, authorization for the NOIP program has been expanded; nuclear-trained surface warfare officers are now covered under NOIP. Annual payment amounts have also increased. The current authorization, which was enacted in 1987 and expired in 1996, provided for up to $12,000 per year in Continuation Pay (COPAY), $7,200 per year for the Annual Incentive Bonus (AIB), and an accession bonus of $6,000.

Research on the effects of NOIP on nuclear officer retention is long overdue. This issue was last studied in 1981 (CNA 81-0130). New legislative authority for NOIP required more current estimates of its effects on retention.

Approach

The approach included: (1) developing a model of retention behavior, and (2) estimating and validating the model.

Data

The primary data source for this research was the Navy's Officer Master File. Cohorts of nuclear-trained officers were assembled and tracked by Social Security Number. Cohorts were categorized by the fiscal year in which officers were commissioned. Complete officer data prior to FY74 was not available. Thus, the first cohort in the data set was the FY74 cohort.

The data set contained 10,357 officers. Only unrestricted line officers were considered; limited duty and warrant officers (149 observations) were dropped from the data set. Censored observations totalling 2,121 officers were also deleted. Officers were censored if: (1) they had not completed their minimum service requirement (MSR) (1,817 observations), or (2) they attrited prior to MSR (304 observations).

From the remaining 8,087 observations, 589 additional officers, many who had prior enlisted service and others who had missing data, were excluded from the final data set. The data set that was used to estimate and validate the models, then, contained 7,498 observations. Approximately 10 percent of the 7,498 observations, or 743 officers, were set aside for model validation.

The end of the minimum service requirement for most nuclear officers in this data set was 5 years after commissioning. For the first cohort, the FY74 cohort, most of the officers made their

initial stay-leave decision in FY79. Similarly, for the last cohort, the FY89 cohort, their initial stay-leave decision occurred in FY94. For FY79 through FY94, the following percentages were computed:

$$\text{CONTRACT Participation Rate} = \frac{\text{\# Making a Decision to Stay in the Navy w / Contract in FY}}{\text{\# Eligible to Make a Stay - Leave Decision in FY}} * 100\%$$

$$\text{NOCONTRACT Participation Rate} = \frac{\text{\# Making a Decision to Stay in the Navy w / o Contract in FY}}{\text{\# Eligible to Make a Stay - Leave Decision in FY}} * 100\%$$

Figure 1 displays the historical CONTRACT and NOCONTRACT participation rates for the 6,755 observations used to estimate the model. The sum of the CONTRACT and NOCONTRACT participation rates is the MSR retention rate (or MSR retention rate is equal to 1 minus the leave rate at MSR).

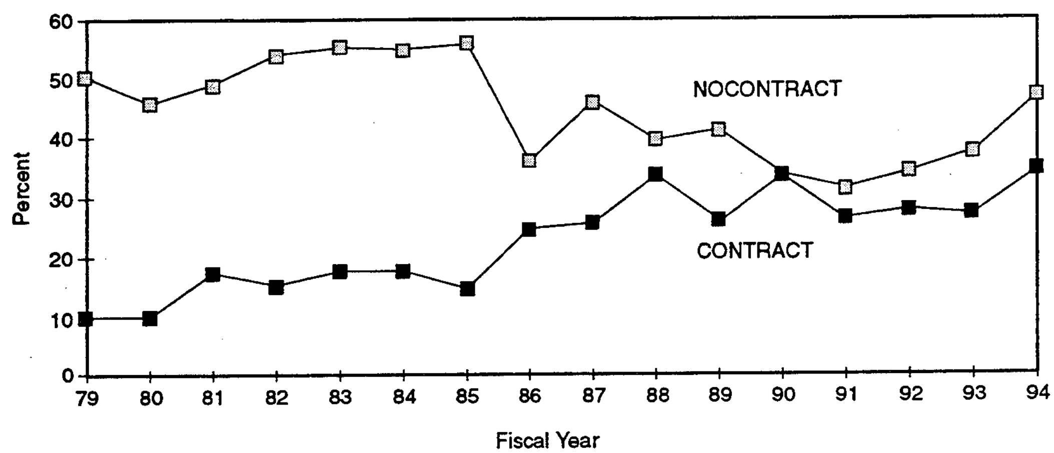

Figure 1. CONTRACT/NOCONTRACT participation rates at MSR by fiscal year.

The 6,755 officers were separated into the two nuclear power communities, submarine and surface warfare. A summary of their MSR decisions is displayed in Table 1. The number in parenthesis is the row percentage. Submarine officers had higher MSR retention rates than surface warfare officers (i.e., 67 and 56 percent, respectively). While staying without a contract was favored over a contractual extension among the stayers, submarine officers were more likely to choose a contract than surface warfare officers (i.e., 36 and 30 percent, respectively).

Table 1

MSR Decision by Officer Community

	Stay w/ Contract	Stay w/o Contract	Leave the Navy	Total
Submarine Officer	1,322 (24%)	2,322 (43%)	1,804 (33%)	5,448
Surface Warfare Officer	217 (17%)	515 (39%)	575 (44%)	1,307
Total	**1,539 (23%)**	**2,837 (42%)**	**2,379 (35%)**	**6,755**

The MSR decision of the 6,755 officers by their accession source is displayed in Table 2. Academy accessions made up 38 percent (2591/6755) of the nuclear community with NROTC and other accession sources almost equally dividing the remainder. Academy accessions have the highest MSR retention rate, followed by NROTC accessions. Regardless of accession source, staying without a contract was favored over a contractual extension by nearly a 2 to 1 margin.

Table 2

MSR Decision by Accession Source

	Stay w/ Contract	Stay w/o Contract	Leave the Navy	Total
U.S. Naval Academy Accession	681 (26%)	1,165 (45%)	745 (29%)	2,591
NROTC Accession	418 (21%)	885 (44%)	712 (35%)	2,015
Other Accession Sources	440 (20%)	787 (37%)	922 (43%)	2,149
Total	**1,539 (23%)**	**2,837 (42%)**	**2,379 (35%)**	**6,755**

Historically, family separation has been a major reason officers give for leaving the Navy. Hence, the number of dependents may influence the stay-leave decision at MSR for these 6,755 officers. Table 3 displays the MSR decision by the number of dependents the officer had at MSR. Most officers were single at MSR, and they had the lowest retention rate, 62 percent. Even when they stayed, single officers were more likely to extend without obligation. In the 1981 study, nuclear submarine officers were more likely to stay at MSR as the number of dependents increased. These data show the same pattern, namely, as the number of dependents increased, the greater the likelihood the officer stayed, and if he stayed, the more likely it was that he obligated.

Table 3

MSR Decision by Number of Dependents

	Stay w/ Contract	Stay w/o Contract	Leave the Navy	Total
No dependents at MSR	538 (17%)	1,406 (44%)	1,216 (38%)	3,160
One dependent at MSR	666 (26%)	1,042 (40%)	892 (34%)	2,600
> Two dependents at MSR	335 (34%)	389 (39%)	271 (27%)	995
Total	**1,539 (23%)**	**2,837 (42%)**	**2,379 (35%)**	**6,755**

Model

Nuclear-trained officers at the end of their minimum service requirement (MSR) can choose to: (1) stay in the Navy under contract, (2) stay in the Navy under no obligation, or (3) leave the Navy.

From these 3 choices, an officer chooses that outcome that maximizes his utility. The utility function for each outcome can be written:

$$U'_C = \theta'_C + \beta M_C + \alpha'_C Z + \gamma'_C$$
$$U'_A = \theta'_A + \beta M_A + \alpha'_A Z + \gamma'_A$$
$$U'_L = \theta'_L + \beta M_L + \alpha'_L Z + \gamma'_L$$

where

U'_C = utility from staying in the Navy at MSR with a contract
U'_A = utility from staying in the Navy at MSR without a contract
U'_L = utility from leaving the Navy at MSR
M_C = vector containing the annualized military pay over the horizon of a COPAY contract
M_A = vector containing the annualized military pay over the horizon of an AIB extension
M_L = vector containing the annualized civilian pay
Z = vector of observable characteristics of the officer (e.g., accession source, dependents)
γ'_C, γ'_A, γ'_L = random error terms

and θ_i, α_i, and β are coefficients.

Because utility is a random function, the probability of choosing to stay in the Navy under contract, choosing to stay in the Navy without a contract, or leaving the Navy is given by:

$$P_i = \frac{\exp\left(\theta_i' + \beta M_i + a_i' Z\right)}{\sum_{j=1}^{m} \exp\left(\theta_j' + \beta M_j + a_j' Z\right)}$$

The complete development of the economic model is found in Appendix A. In this general form of the logit model, M_i is the vector of attribute values for the *ith* choice, which includes the compensation variable. Note that the coefficient on M_i is constant across the choices. The officer-specific variables are described in Z and its coefficient varies across the choices. A similar model was used by Goldberg and Warner (CRC 476) to describe reenlistment and extension behavior. Maximum likelihood methods were used to estimate the parameters of this model and the findings are described in the Results section.

Model parameters were estimated for (1) submarine officers, (2) surface warfare officers, and (3) submarine and surface warfare officers combined. In Table 4, the variables are defined, the vector of which they are components is identified, and their mean values are presented.

The compensation variables were constructed as follows. Submarine officers' MILPAYC, for example, is the annualized value of RMC plus SUBPAY plus COPAY over the contract horizon. (Note: Prior to 1981, SUBPAY was only paid while serving on a submarine. From 1981 onward, SUBPAY was paid as long as an officer was sub-qualified.) A 4-year horizon was chosen because it was common throughout the years of observation. In October 1985, the 3- and 5-year COPAY options were added. COPAY is paid in equal annual payments, and the amount of the annual payment does not depend on the contract length. Currently, the annual COPAY payment is $10,000 for each year of a 3-, 4-, or 5-year contract. Surface warfare officers' MILPAYC was constructed in the same way, but does not include SUBPAY. A discount rate of 10 percent was assumed (r = .10). For submarine officers,

$$MILPAYC = \frac{\sum_{i=t+1}^{t+4} (RMC_i + SUBPAY_i + COPAY_i)\,(1+r)^{t-i+1}}{\sum_{i=t+1}^{t+4} (1+r)^{t-i+1}}$$

For surface warfare officers, MILPAYC is given by:

$$MILPAYC = \frac{\sum_{i=t+1}^{t+4} (RMC_i + COPAY_i)\,(1+r)^{t-i+1}}{\sum_{i=t+1}^{t+4} (1+r)^{t-i+1}}$$

Table 4

Variable Descriptions and Mean Values

Variables *Vector*		Description	Submarine Model $N = 5{,}448$	Surface Model $N = 1{,}307$	Combined Model $N = 6{,}755$
CONTRACT	= =	1, if stay in the Navy at MSR w/ a 3-, 4-, or 5-year contract 0, otherwise	.243	.166	.228
NOCONTRACT	= =	1, if stay in the Navy at MSR wo/ a 3-, 4-, or 5-year contract 0, otherwise	.426	.394	.420
LEAVE	= =	1, if leave in the Navy at MSR 0, otherwise	.331	.440	.352
MILPAYC M_C *vector*	=	Annualized military pay over a 4-year horizon; military pay includes RMC, SUBPAY (submariners only) and COPAY (FY89 $s)	$53,654	$48,853	$52,725
MILPAYA M_A *vector*	=	Annualized military pay over a 1-year horizon; military pay includes RMC, SUBPAY (submariners only) and AIB (FY89 $s)	$51,550	$46,755	$50,622
CIVPAY M_L *vector*	=	Annualized civilian earnings over a 4-year horizon (FY89 $s)	$24,575	$24,382	$24,538
COPAY345 M_C *vectors*	= =	1, if MSR occurred in FY86 or later 0, otherwise	.604	.611	.606
SUB *Z vector*	= =	1, if submarine officer 0, otherwise			.806
ACA *Z vector*	= =	1, if U.S. Naval Academy accession 0, otherwise	.374	.423	.384
NROTC *Z vector*	= =	1, if NROTC accession 0, otherwise	.292	.324	.298
WHITE *Z vector*	= =	1, if officer is white 0, otherwise	.952	.930	.948
DEPEND *Z vector*	=	Number of dependents at MSR	.752	.640	.731

The major difference in the construction of MILPAYC and MILPAYA was the horizon. While MILPAYC assumed a 4-year horizon, MILPAYA was defined over a 1-year horizon. All nuclear-qualified officers on active-duty at the end of the fiscal year received the AIB payment.

The civilian earnings variable, CIVPAY, assumed a 4-year horizon. Each year's civilian earnings was determined from the following equation:[1]

$$
\begin{aligned}
LnY = {} & 9.7060 + .0701 * MILEXP - .0014 * MILEXP^2 + .0785 * CIVEXP - .0013 * CIVEXP^2 \\
& - .0025 * MILEXP * CIVEXP - .2366 * NONWHITE + .1053 * BAPLUS
\end{aligned}
$$

where

LnY	=	natural logarithm of annual earnings in 1989 dollars
MILEXP	=	years of military experience
CIVEXP	=	years of civilian experience
NONWHITE	=	1, if the veteran was nonwhite; = 0, otherwise
BAPLUS	=	1, if the veteran had more than a 4-year degree; = 0, otherwise

The equation was estimated using a cross-sectional sample of veterans from the Census Bureau's 1990 Public Use Microdata Samples (PUMS). The sample included only male, college-graduate veterans in engineering and managerial occupations. CIVPAY does not include bonuses or benefits. Comparing CIVPAY to basic pay for an officer with 5 years of service would be more meaningful. In 1989 dollars, basic pay for an officer with 5 years of service was approximately $29,000.

As pointed out previously, in October 1985, a 3- and 5-year contract option in addition to the original 4-year contract was made available. Over the observation period, this was the only significant change in NOIP administration affecting the officers at the MSR decision point. The dichotomous variable, COPAY345, captures this change in the choice to stay in the Navy.

Results

Four separate models were estimated: (1) submarine officers only, (2) surface warfare officers only, (3) submarine and surface warfare officers combined, and (4) a combined model with the additional SUB variable. The results are displayed in Table 5. In general, the coefficients are statistically significant at the 5 percent level. At the bottom of the table, the Chi-square (χ^2) statistic is one measure of the model's goodness-of-fit. In each model, χ^2 is well above the level of statistical significance at the 5 percent level.

The model coefficients are not the partial derivatives of P_C or P_A, with respect to a particular variable. The partial derivatives, the effect of a particular variable on P_C or P_A, are found in Appendix A. The sign on the coefficient, however, indicates whether the variable had a positive or negative effect on P_C or P_A. Within each of the four models, the coefficients on the compensation variables, MILPAYC, MILPAYA, and CIVPAY, are the same, and the positive sign indicates that <u>pay matters</u> in retention decisions. The compensation coefficients for the combined models were

[1] This earnings equation was estimated by Systems Analytics Group, Inc. under contract to the Navy Personnel Research and Development Center.

close to that of the submarine model because of the size of submarine community relative to the surface community. The other choice variable, COPAY345, had a significant, positive impact on retention. The models suggest that eliminating the 3- and 5-year contract option would lead to pre-FY86 CONTRACT participation rates.

Table 5

Estimates of Models' Parameters
(t = statistics in parenthesis)

Variables Describing M	Submarine Model	Surface Model	Combined Model 1	Combined Model 2
MILPAYC, MILPAYA, CIVPAY	.0000417 (2.790)	.0002015 (4.730)	.0000793 (8.345)	.0000536 (3.885)
COPAY 345	0.960 (13.078)	1.196 (6.522)	0.973 (14.342)	0.987 (14.500)
Variables Describing Z (CONTRACT choice)				
Constant	-3.373 (-6.533)	-9.074 (-7.080)	-4.738 (-13.768)	-4.293 (-10.203)
ACA	0.624 (5.913)	0.990 (4.393)	0.576 (6.693)	0.680 (7.231)
NROTC	0.256 (2.451)	0.224 (0.924)	0.156 (1.742)	0.239 (2.520)
WHITE	0.618 (3.004)	1.669 (3.937)	0.891 (5.174)	0.736 (4.039)
DEPEND	0.401 (9.364)	0.376 (3.920)	0.396 (10.219)	0.394 (10.128)
SUB				0.430 (3.800)
Variables Describing Z (NOCONTRACT choice)				
Constant	-1.424 (-3.016)	-6.203 (-5.368)	-2.653 (-8.719)	-2.020 (-5.379)
ACA	0.454 (4.764)	0.368 (2.133)	0.363 (4.918)	0.456 (5.493)
NROTC	0.285 (3.152)	0.113 (0.655)	0.188 (2.541)	0.260 (3.262)
WHITE	0.270 (1.622)	1.469 (4.339)	0.564 (4.244)	0.420 (2.898)
DEPEND	0.075 (1.900)	0.032 (0.400)	0.065 (1.838)	0.068 (1.909)
SUB				0.145 (1.501)
Log Likelihood Function	-5642.102	-1292.036	-6938.697	-6930.462
Chi-square	408.18	126.62	563.29	579.76

The officer-specific variables described in Z are interpreted as correlates of taste. There was no a priori hypothesis about the signs of the coefficients, which vary across the choices. Because of the structure of this logit model, coefficients on Z for the third choice, leave the Navy, were normalized to zero. Hence, Table 5 contains only the coefficients for the remaining two choices.

The significant, positive ACA coefficients in both nuclear communities in both the CONTRACT and NOCONTRACT choices indicate that relative to the omitted accession group (other accession sources), Naval Academy accessions had higher retention propensities. In the submarine community, NROTC accessions also had higher retention propensities than the omitted group (significant, positive NROTC coefficients in both the COPAY and AIB choices). The coefficients on NROTC in the surface model are not statistically significant implying that (relative to the omitted group) being an NROTC accession did not enhance the likelihood of staying in the Navy.

In Table 3, there were indications that the number of dependents affected an officer's MSR stay-leave decision. The positive DEPEND coefficients in both nuclear communities in both the CONTRACT and NOCONTRACT choices indicate that the greater the number of dependents an officer had at MSR the greater was the likelihood he would stay in the Navy. Moreover, the impact of dependents was statistically significant for both communities in the CONTRACT choice. That is, the greater the number of dependents the higher the likelihood of staying in the Navy under contract.

Another measure of the models' goodness-of-fit was their ability to predict the stay-leave propensities of the validation sample. Recall that 743 observations were set aside for this validation. The results are presented in Table 6. The models' predictions are in italics. All the models underpredict leaving the Navy. The submarine model overpredicts staying without a contract and underpredicts staying with a contract. Because of the size of the submarine community relative to the surface warfare community, the combined models also overpredict staying without a contract and underpredict staying with a contract. The surface model, on the other hand, overpredicts staying with a contract and underpredicts staying without a contract.

Table 6

Models' Validation: Actual vs. Predicted

Outcome	Submarine Model	Surface Model	Combined Model 1	Combined Model 2
CONTRACT	.259	.135	.234	.234
	.244	*.175*	*.232*	*.231*
NOCONTRACT	.382	.392	.384	.384
	.425	*.382*	*.416*	*.416*
LEAVE	*.360*	.473	.382	.382
	.331	*.443*	*.352*	*.352*

Elasticity formulas that measure P_C, P_A, and P_L responses to changes in MILPAYC (M_C) and MILPAYA (M_A) were developed in Appendix A. Those formulas do not measure P_C, P_A, and P_L responses to changes in annual payments of COPAY and AIB. To quantify the responsiveness of P_C, P_A, and P_L to changes in annual COPAY and AIB payments, the models were exercised twice. The first simulation determined P_C, P_A, and P_L with a 10 percent increase in annual COPAY payments. A percentage change between these new rates and the current, observed rates was calculated. Dividing this percentage by 10 percent produced the P_C, P_A, and P_L elasticities with respect to a 10 percent change in annual COPAY payments. These elasticities are found in Table 7. The second simulation determined P_C, P_A, and P_L with a 10 percent increase in annual AIB payments. Elasticities with respect to a 10 percent change in annual AIB payments were then computed. These elasticities are found in Table 8.

Table 7

P_C, P_A, and P_L Elasticities with Respect to a 10 Percent Change in COPAY

Elasticity	Submarine Model	Surface Model	Combined Model 1	Combined Model 2
$e_{LC} = e_{AC}$	-0.112	-0.394	-0.201	-0.135
e_{CC}	0.258	1.442	0.507	0.341

Table 8

P_C, P_A, and P_L Elasticities with Respect to a 10 Percent Change in AIB

Elasticity	Submarine Model	Surface Model	Combined Model 1	Combined Model 2
$e_{LA} = e_{CA}$	-0.108	-0.477	-0.203	-0.135
e_{AA}	0.165	0.812	0.314	0.212

From the elasticity formulas in Appendix A, it was shown that e_{LC} equals e_{AC} and e_{LA} equals e_{CA}; increases in COPAY and AIB draw from the other two choices equally in percentage terms. For example, for submarine officers, a 10 percent increase in COPAY increases the probability of staying in the Navy with a contract by 2.58 (e_{CC}*10) percent while decreasing both the probability of staying in the Navy without a contract and the probability of leaving the Navy by 1.12 (e_{LC}*10) percent. Recall that the retention rate is 1 minus P_L; thus, -e_{LC} is the retention rate elasticity with respect to changes in COPAY, and -e_{LA} is the retention rate elasticity with respect to changes in AIB. Within the same model, the 10 percent increase in annual COPAY and AIB payments had nearly the same effect on retention. For example, in the submarine model, the 10 percent increase in annual COPAY payments increased the retention rate by 1.12 (-e_{LC}*10) percent; the 10 percent increase in annual AIB payments increased the retention rate by 1.08 (-e_{LA}*10) percent. In the surface model, the 10 percent increase in annual COPAY and AIB payments increased the retention

rate by 3.94 (-e_{LC}*10) and 4.77 (-e_{LA}*10) percent, respectively. Nuclear surface warfare officers were more responsive to compensation changes than submarine officers.

While increases in COPAY and AIB had nearly the same effect on overall retention, increases in COPAY had larger effects on the probability of staying in the Navy with a contract than increases in AIB had on the probability of staying in the Navy without a contract. This was true across all models. For example, in the submarine model, a 10 percent increase in annual COPAY payments increased the probability of staying in the Navy with a contract by 2.58 (e_{CC}*10) percent; a 10 percent increase in annual AIB payments increased the probability of staying in the Navy without a contract by 1.65 (e_{AA}*10) percent. In the surface model, a 10 percent increase in annual COPAY payments increased the probability of staying in the Navy with a contract by 14.42 (e_{CC}*10) percent. The increase in the probability of staying in the Navy without a contract caused by a 10 percent increase in annual AIB payments was 8.12 (e_{AA}*10) percent. In the 1981 study, increases in COPAY also had a greater effect on staying in the Navy with a contract than increases in AIB had on staying in the Navy without a contract.

Conclusions

Most retention-compensation research, including the first nuclear submarine officer study in 1981, finds that "pay does matter." While the retention bonus elasticities found in these models are relatively small, their statistical significance also points to the fact that pay matters. There were other similar findings between this model and the earlier one. For example, both studies report that an increase in COPAY increases probability of staying in the Navy with a contract at MSR more than an increase in AIB increases probability of staying in the Navy without a contract at MSR. Second, Naval Academy accessions were found to have higher propensities to stay in the Navy at MSR than other source accessions. Finally, both studies report that the greater the number of dependents an officer has at MSR the higher is the likelihood he will stay in the Navy and the higher is the likelihood he will stay with a contract. These models were used to simulate a 10 percent increase in COPAY and a 10 percent increase in AIB, but they can also be used to assess the retention and cost impacts of other alternative NOIP retention bonus strategies.

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*Maddala, G. S. *Limited-dependent and qualitative variables in econometrics*. London: Cambridge University Press, 1983.

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*Maddala is referenced in Appendix A.

Appendix A

Nuclear Officer Retention Model

Nuclear-trained officers at the end of their minimum service requirement (MSR) can choose: (1) to stay in the Navy under contract (COPAY), (2) to stay in the Navy under no obligation (AIB), or (3) leave the Navy (Leave). From these 3 choices, an officer chooses that outcome that maximizes his utility. In Goldberg and Warner (1982), the utility function is the sum of two vectors. The first vector contains the annualized income stream associated with that outcome and other outcome characteristics (e.g., contract length). The second vector represents the officer's taste for military service. Mathematically,

$$U_{Cj} = M_{Cj} + \delta_C$$
$$U_{Aj} = M_{Aj} + \delta_A$$
$$U_{Lj} = M_{Lj} + \delta_L$$

where

U_{Cj} = utility from staying in the Navy at MSR with a contract

U_{Aj} = utility from staying in the Navy at MSR without a contract

U_{Lj} = utility from leaving the Navy at MSR

M_{Cj} = vector containing the annualized military pay over the horizon of a COPAY contract

M_{Aj} = vector containing the annualized military pay over the horizon of an AIB extension

M_{Lj} = vector containing annualized civilian pay

δ_C, δ_A, and δ_L = tastes for military service

Tastes for military service may vary systematically with individual characteristics such as accession source, race and number of dependents. Let δ_i be explained by a vector, Z_j, of observable characteristics for the *jth* individual and a random error, γ_i. Then

$$U_{Cj} = M_{Cj} + \alpha_C Z_j + \gamma_{Cj} \qquad (1)$$

$$U_{Aj} = M_{Aj} + \alpha_A Z_j + \gamma_{Aj} \qquad (2)$$

$$U_{Lj} = M_{Lj} + \alpha_L Z_j + \gamma_{Lj} \qquad (3)$$

Z_j = vector of observable characteristics for the *jth* individual

α_A, α_C, α_L = vector of coefficients
γ_{Cj}, γ_{Aj}, γ_{Lj} = random error terms

For ease of exposition, the subscript j will be left off for the remainder of this section. From Maddala (1983), let the random error terms, γ_C, γ_A, and γ_L be linear transformations of independently and identically distributed (IID) standardized extreme value random variables, $\gamma_i^{'}$, i.e.,

$$\gamma_i = \theta_i + \theta_o \gamma_i^{'}, \quad \theta_o > 0, \quad i = C, A, L$$

The cumulative distributive function (CDF) of the residuals, $\gamma_i^{'}$, is $F(\gamma_i^{'} < \gamma) = \exp(-e^{-\gamma})$. The probability density function (PDF) is $f(\gamma_i^{'}) = \exp(-\gamma_i^{'} - e^{-\gamma_i^{'}})$.

Substituting for γ_i into equations (1), (2), and (3) yields

$$U_C = M_C + \alpha_C Z + \theta_C + \theta_o \gamma_C^{'} \qquad (4)$$

$$U_A = M_A + \alpha_A Z + \theta_A + \theta_o \gamma_A^{'} \qquad (5)$$

$$U_L = M_L + \alpha_L Z + \theta_L + \theta_o \gamma_L^{'} \qquad (6)$$

Dividing these utility functions by θ_o yields the following normalized utilities:

$$U_C^{'} = \theta_C^{'} + \beta M_C + \alpha_C^{'} Z + \gamma_C^{'} \qquad (7)$$

$$U_A^{'} = \theta_A^{'} + \beta M_A + \alpha_A^{'} Z + \gamma_A^{'} \qquad (8)$$

$$U_L^{'} = \theta_L^{'} + \beta M_L + \alpha_L^{'} Z + \gamma_L^{'} \qquad (9)$$

where

$\theta_i^{'} = \theta_i / \theta_o, i = C, A, L$
$\alpha_i^{'} = \alpha_i / \theta_o, i = C, A, L$
$\beta = 1 / \theta_o$

Note that the officer's utility maximization problem is unaffected by dividing the originial utility functions, U_i, by θ_o. Also, U_i and U_i' are not directly observable. Instead, choice i is observable and is defined as

$$Y_i = 1 \text{ if } U_i' = Max(U_1', U_2', \ldots, U_m')$$
$$Y_i = 0 \text{ otherwise}$$

The condition $U_i' = Max(U_1', U_2', \ldots, U_m')$ implies

$$U_i' > U_j' \text{ for all } j \neq i$$

or

$$\theta_i' + \beta M_i + \alpha_i' Z + \gamma_i' > \theta_j + \beta M_j + \alpha_j' Z + \gamma_j' \text{ for all } j \neq i$$

or

$$\gamma_j' < \gamma_i' + \theta_i' + \beta M_i + \alpha_i' Z - \theta_j' - \beta M_j - \alpha_j' Z \text{ for all } j \neq i$$

Because γ_i are IID with CDF defined above, it follows that

$$\Pr ob(Y_i = 1) = \Pr ob(\gamma_j' < \gamma_i' + \theta_i' + \beta M_i + \alpha_i' Z - \theta_j' - \beta M_j - \alpha_j' Z) \text{ for all } j \neq i$$

$$= \int_{-\infty}^{\infty} \prod_{j \neq i} F(\gamma_i' + \theta_i' + \beta M_i + \alpha_i' Z - \theta_j' - \beta M_j - \alpha_j' Z) f(\gamma_i') d\gamma_i \qquad (10)$$

From the CDF and PDF defined above,

$$\prod_{j \neq i} F(\gamma_i' + \theta_i' + \beta M_i + \alpha_i' Z - \theta_j' - \beta M_j - \alpha_j' Z) f(\gamma_i') =$$

$$\prod_{j \neq i} \exp(-\exp(-\gamma_i' - \theta_i' - \beta M_i - \alpha_i' Z + \theta_j' + \beta M_j + \alpha_j' Z)) \exp(-\gamma_i' - \exp(-\gamma_i')) =$$

$$(11)$$

$$\exp\left[\gamma_i' - \exp(-\gamma_i')\left(1 + \sum_{j \neq i} \frac{\exp(\theta_j' + \beta M_j + \alpha_j' Z)}{\exp(\theta_i' + \beta M_i + \alpha_i' Z)}\right)\right]$$

Let

$$\lambda_i = \ln\left(1 + \sum_{j \neq i} \frac{\exp(\theta_j' + \beta M_j + \alpha_j' Z)}{\exp(\theta_i' + \beta M_i + \alpha_i' Z)}\right) = \ln\left(\sum_{j=1}^{m} \frac{\exp(\theta_j' + \beta M_j + \alpha_j' Z)}{\exp(\theta_i' + \beta M_i + \alpha_i' Z)}\right) \qquad (12)$$

and substituting into (11) yields

$$\prod_{j\neq i} F(\gamma_i' + \theta_i' + \beta M_i + \alpha_i' Z - \theta_j' - \beta M_j - \alpha_j' Z) f(\gamma_i') = \exp(-\gamma_i' - e^{-(\gamma_i' - \lambda_i')}) \tag{13}$$

Substituting (13) into (10) yields

$$\int_{-\infty}^{\infty} \exp(-\gamma_i' - e^{-(\gamma_i' - \lambda_i')}) d\gamma_i' = \exp(-\lambda_i') \int \exp(-\gamma_i^* - e^{-\gamma_i^*}) d\gamma_i^* \quad \text{where } \gamma_i^* = \gamma_i' - \lambda_i$$

$$= \exp(-\lambda_i') \tag{14}$$

$$= \frac{\exp(\theta_i' + \beta M_i + \alpha_i' Z)}{\sum_{j=1}^{m} \exp(\theta_j' + \beta M_j + \alpha_j' Z)} \tag{15}$$

Thus, because utility is a random function and given the distributional assumption on γ_i', the probability of choosing to stay in the Navy under contract (P_C), choosing to stay without a contract (P_A), or leaving the Navy (P_L) is given by:

$$P_i = \frac{\exp(\theta_i' + \beta M_i + \alpha_i' Z)}{\sum_{j=1}^{m} \exp(\theta_j' + \beta M_j + \alpha_j' Z)} \tag{16}$$

In this general form of the logit model, M_i is the vector of values of the attributes of the *ith* choice, which includes the compensation variable. Note that the coefficient on M_i is constant across the choices. The officer-specific variables are described in Z and its coefficient is varies across the choices.

The effects on P_i in equation (16) above caused by changes in compensation are shown below. For changes in P_i resulting from changes in M_C, differentiating equation (16) with respect to M_C yields:

$$\frac{\partial P_L}{\partial M_C} = -\beta P_L P_C \Rightarrow \varepsilon_{LC} = \frac{\partial P_L}{\partial M_C}\frac{M_C}{P_L} = -\beta P_L P_C \frac{M_C}{P_L} = -\beta P_C M_C$$

$$\frac{\partial P_A}{\partial M_C} = -\beta P_A P_C \Rightarrow \varepsilon_{AC} = \frac{\partial P_A}{\partial M_C}\frac{M_C}{P_A} = -\beta P_A P_C \frac{M_C}{P_A} = -\beta P_C M_C = \varepsilon_{LC}$$

$$\frac{\partial P_C}{\partial M_C} = \beta P_C(1-P_C) \Rightarrow \varepsilon_{CC} = \frac{\partial P_C}{\partial M_C}\frac{M_C}{P_C} = \beta P_C(1-P_C)\frac{M_C}{P_C} = \beta(1-P_C)M_C$$

Since β is positive, increasing M_C increases P_C at the expense of P_A and P_L, i.e., NOCONTRACT participation falls and overall retention rises (number of leavers declines). ε_{ij} is the elasticity of P_i with respect to M_j.

Similarly, an increase in M_A, increases NOCONTRACT participation, lowers CONTRACT participation, and increases retention. For changes in P_i resulting from changes in M_A, differentiating equation (16) with respect to M_A yields:

$$\frac{\partial P_L}{\partial M_A} = -\beta P_L P_A \Rightarrow \varepsilon_{LA} = \frac{\partial P_L}{\partial M_A}\frac{M_A}{P_L} = -\beta P_L P_A \frac{M_A}{P_L} = -\beta P_A M_A$$

$$\frac{\partial P_C}{\partial M_A} = -\beta P_C P_A \Rightarrow \varepsilon_{CA} = \frac{\partial P_C}{\partial M_A}\frac{M_A}{P_C} = -\beta P_C P_A \frac{M_A}{P_C} = -\beta P_A M_A = \varepsilon_{LA}$$

$$\frac{\partial P_A}{\partial M_A} = \beta P_A(1-P_A) \Rightarrow \varepsilon_{AA} = \frac{\partial P_A}{\partial M_A}\frac{M_A}{P_A} = \beta P_A(1-P_A)\frac{M_A}{P_A} = \beta(1-P_A)M_A$$

Table A-1 displays these elasticities which were evaluated at the sample means. The elasticity of P_C with respect to MILPAYC, ε_{CC}, and the elasticity of P_A with respect to MILPAYA, ε_{AA}, are 1.694 and 1.234, respectively. The 1981 report, which focused only on submarince officers, found an elasticity of extending with respect to military pay of 1.4.

Caution should be used in the interpetation of the elasticities in table A-1. These are not the retention rate elasticities with respect to COPAY or AIB. Recall that MILPAYC was the annualized sum of RMC, SUBPAY (for submarine officers), and COPAY while MILPAYA was the annualized sum of RMC, SUBPAY (for submarine officers), and AIB. So, a 10 percent change in MILPAYC implies a 10 percent change in RMC, SUBPAY (for submarine officers), and COPAY over the 4-year horizon. Hence, the elasticities in table A-1 are the percentage change in retention rate divided by the percentage change in RMC, SUBPAY (for submarine officers), and COPAY or AIB over the appropriate horizon.

Table A-1
P_L, P_A, and P_C Elasticities with Respect to MILPAYC (M_C) and MILPAYA (M_A)

Elasticity	Submarine Model	Surface Model	Combined Model 1	Combined Model 2
ε_{LC}	-0.544	-1.634	-0.953	-0.644
ε_{AC}	-0.544	-1.634	-0.953	-0.644
ε_{CC}	1.694	8.21	3.228	2.182
ε_{LA}	-0.916	-3.712	-1.686	-1.14
ε_{CA}	-0.916	-3.712	-1.686	-1.14
ε_{AA}	1.234	5.709	2.328	1.574

Distribution List

Chief of Naval Personnel (PERS-00), (PERS-00b), (PERS-00H), (PERS-00W), (PERS-01JJ), (PERS-2), (PERS-23), (PERS-24)
Director, Army Research Institute (PERI-ZT), Alexandria, VA
Operations and Support Directorate, Armstrong Laboratory (AL/DO), (AL/HR-DOKL Technical Library), Brooks Air Force Base, TX
President, Naval War College (Code E-1121)
Superintendent, Naval Postgraduate School
Director of Research, U.S. Naval Academy
Center for Naval Analyses, Acquisition Unit
Pentagon Library
Defense Technical Information Center (4)